

Exercício 4.3.1

$$\left\{ \begin{array}{l} a = (10,32 \pm 0,05) \text{ cm} \\ b = (64,27 \pm 0,05) \text{ cm} \end{array} \right. \quad \sigma_a = \sigma_b = \sigma = 0,05 \text{ cm}$$

$$\left\{ \begin{array}{l} P = 2(a + b) \quad \Rightarrow \quad \sigma_P = 2\sqrt{\sigma_a^2 + \sigma_b^2} = 2\sigma\sqrt{2} \\ P = 2 \times 74,59 = 149,18 \quad \Rightarrow \quad \sigma_P = 0,141 \quad \Rightarrow \quad \boxed{P \pm \sigma_P = (149,18 \pm 0,14) \text{ cm}} \end{array} \right.$$

$$\left\{ \begin{array}{l} A = a \times b \quad \Rightarrow \quad \frac{\sigma_A}{A} = \sqrt{\left(\frac{\sigma_a}{a}\right)^2 + \left(\frac{\sigma_b}{b}\right)^2} = \sigma\sqrt{\frac{1}{a^2} + \frac{1}{b^2}} \\ A = 663,27 \quad \Rightarrow \quad \sigma_A = \sigma\sqrt{a^2 + b^2} = 005 \times 65,09 = 3,25 \quad \Rightarrow \quad \boxed{A \pm \sigma_A = (663,3 \pm 3,3) \text{ cm}^2} \end{array} \right.$$

$\Rightarrow$ ou $\boxed{A \pm \sigma_A = (663 \pm 3) \text{ cm}^2}$

Exercício 4.3.2

$$\theta = (22, 3 \pm 0, 5)^{\circ} = (0, 3892 \pm 0, 0087) \text{ rad} \quad \sigma_{\theta} = 0, 0087 \text{ rad}$$

$$\left\{ \begin{array}{l} y = \text{sen } \theta \quad \Rightarrow \quad \sigma_y = \cos \theta \, \sigma_\theta \\ y = 0,379456 \quad \Rightarrow \quad \sigma_y = 0,9252 \times 0,0087 = 0,008 \quad \Rightarrow \quad \boxed{y \pm \sigma_y = (0,379 \pm 0,008)} \end{array} \right.$$

Exercício 4.3.3

$$r = (10,2 \pm 0,3) \text{ cm} \qquad \sigma_r = 0,3 \text{ cm}$$

$$\left\{ \begin{array}{l} A = \pi r^2 \quad \Rightarrow \quad \sigma_A = 2\pi r \sigma_r \\ A = 326,85 \quad \Rightarrow \quad \sigma_A = 19,2 \quad \Rightarrow \quad \boxed{A \pm \sigma_A = (327 \pm 19) \text{ cm}^2} \end{array} \right.$$

Exercício 4.3.4

$$\left\{ \begin{array}{lll} L(\text{trena}) = L_t = 1 \text{ cm} & \Rightarrow \sigma_t = \frac{L_t}{3} \approx 0,3 \text{ cm} = 3 \text{ mm} & \Rightarrow \boxed{\sigma_L = \frac{\sigma_t}{\sqrt{40}} = \frac{3}{\sqrt{40}} \approx 0,5 \text{ mm}} \\ L(\text{régua}) = L_r = 1 \text{ mm} & \Rightarrow \sigma_r = \frac{L_r}{3} = 0,5 \text{ mm} & \Rightarrow \boxed{\sigma_L = \sigma_r = 0,5 \text{ mm}} \end{array} \right.$$

Exercício 4.3.5

x	41.	39.	40.	39.	40.
y	50.	52.	51.	51.	49.

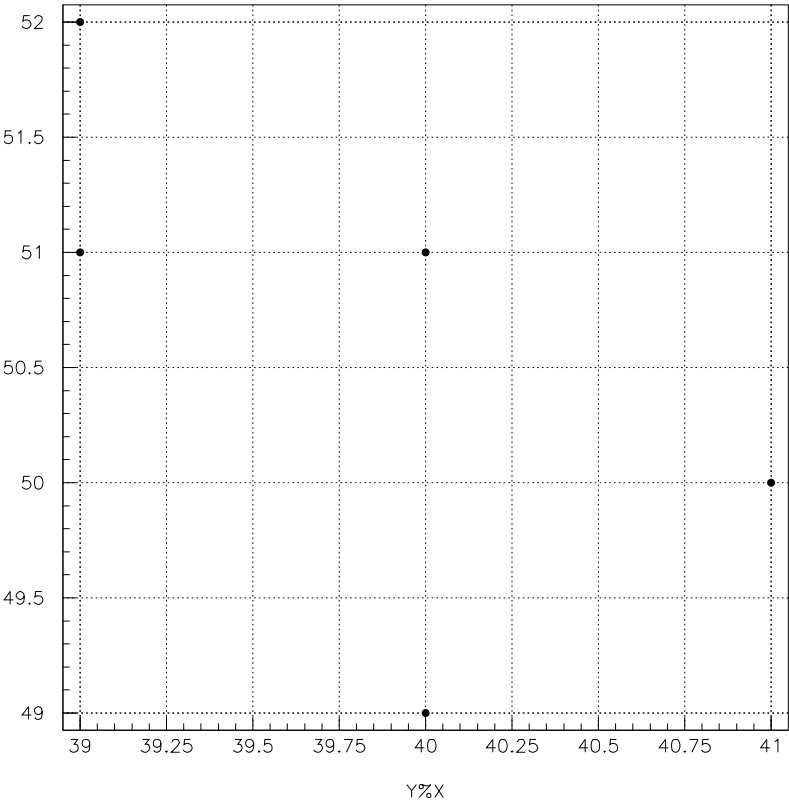
xm	sig_xx	sig_x
39.8	0.56	0.748

ym	sig_yy	sig_y
50.6	1.04	1.0198

sig_xy	r
-0.48	-0.62897

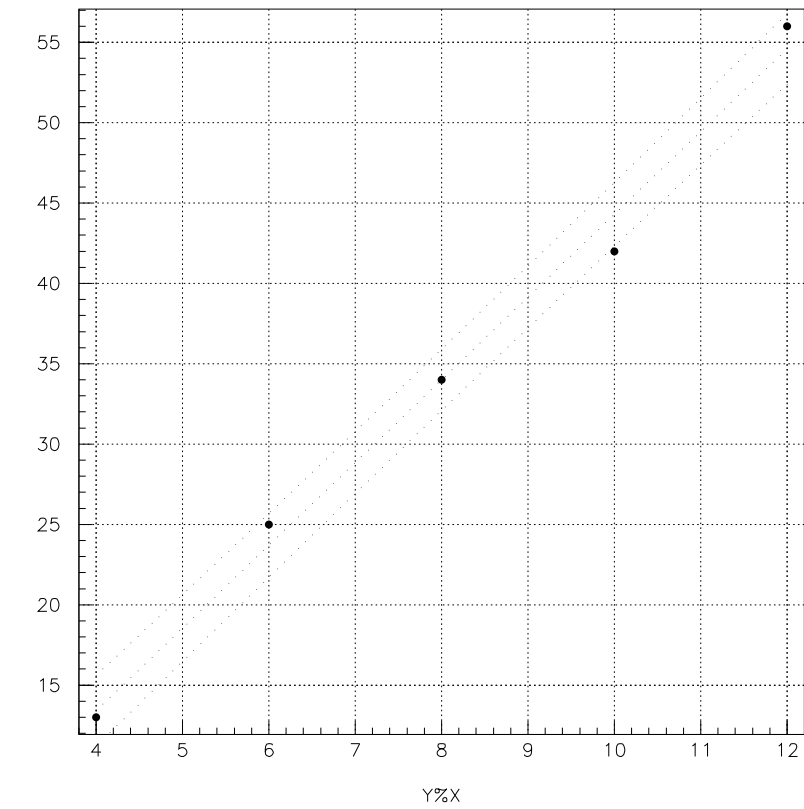
sm = xm + ym	sig_sm	s_ref
90.4	0.4	90.

discrepancia	compatibilidade
sm - s_ref	sm - s_ref /sig_sm
0.3999939	1.1



Exercício 4.3.6

x=t(s)		y=s(cm)			
4.		13.			
6.		25.			
8.		34.			
10.		42.			
12.		56.			
y = v x + b					
sigx	sigy	sigxy(cov)	r(corr)	r^2	
2.828	14.629	41.20	0.99574	0.99150	
a	sig_a	erro	b	sigb	e_y
5.15	0.275	0.053	-7.2	2.337	1.742
v +- sig_v					
(5.15 +- 0.28) cm/s					



Exercício 4.3.7

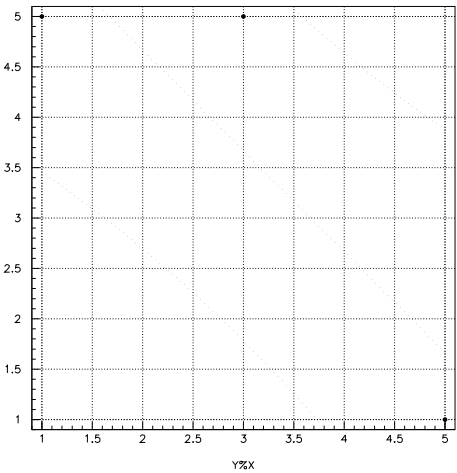
a)

x	y
1.	5.
3.	5.
5.	1.

$y = a x + b$

sigx	sigy	sigxy(cov)	r(corr)	r^2
1.633	1.886	-2.6667	-0.866	0.75

a	siga	b	sigb	e_y
-1.00	0.58	6.67	1.97	1.63



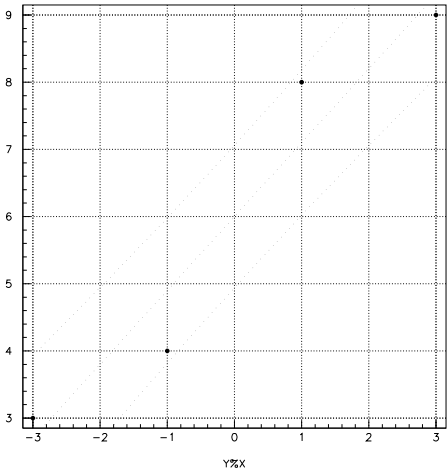
b)

x	y
-3.	3.
-1.	4.
1.	8.
3.	9.

$y = a x + b$

sigx	sigy	sigxy(cov)	r(corr)	r^2
2.236	2.549	5.5	0.965	0.93

a	siga	b	sigb	e_y
1.10	0.212	6.00	0.47	0.95



Exercício 4.3.8

x=P(mm Hg)	y=T(C ⁻¹)
65.	-20.
75.	17.
85.	42.
95.	84.
105.	127.

$$y = a x + b$$

sigx	sigy	sigxy(cov)	r(corr)	r ²
14.142	51.26	722.0	0.99596	0.99194

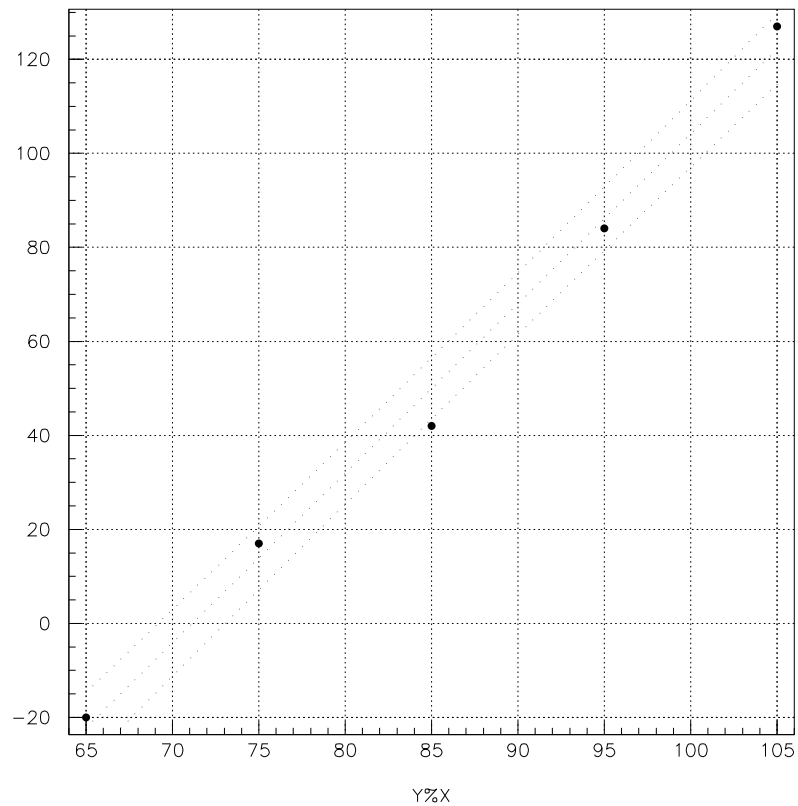
a	sig_a	b	sig_b	e_y
3.61	0.19	-256.85	16.19	5.94

T0	sig_T0	T0_ref
-256.85	16.19	-273.15

discrepancia	compatibilidade
T0 - T0_ref	T0 - T0_ref /sig_T0
16.30	1.0

$$T = 3.61 \times P - 256.85$$

$$P = 80 \text{ mm Hg} \Rightarrow (32 \pm 6) ^\circ\text{C}$$



Exercício 4.3.9

freq (10 ¹⁴ Hz)	potencial (V)
5.19	0.74
5.49	0.84
6.88	1.41
7.41	1.61
8.22	1.98

$$y \text{ (V)} = a \times (10^{15} \text{ Hz}) + b$$

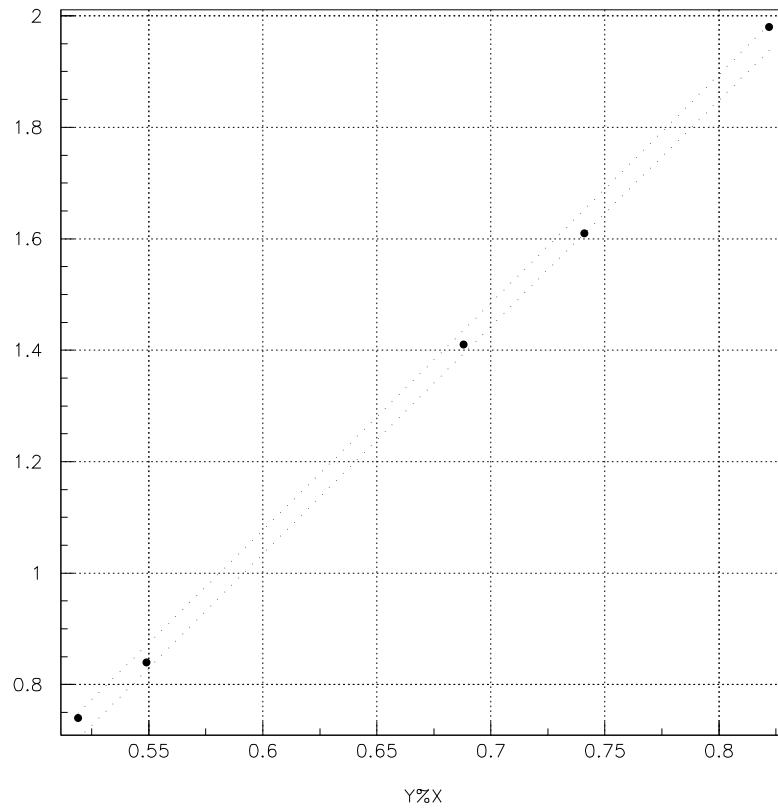
$$a = h/e = 4.1356692 \times 10^{-15} \text{ eV.s}$$

sigx	sigy	sigxy(cov)	r(corr)	r ²
0.1146462	0.4678718	0.0536132	0.9995052	0.9990107

a	sigma	b	sigb	e_y
4.0789843	0.0741087	-1.3916297	0.0499217	0.0189983

h/e (10 ⁻¹⁵ eV.s)	sig_h/e	erro (%)
4.08	0.07	1.8

discrepancia	compatibilidade
h/e - h/e_ref	h/e - h/e_ref /sig_h/e
0.057	0.8



Exercício 4.3.10

$$S(a) = \sum_{i=1}^N (y_i - ax_i)^2 \quad \Rightarrow \quad \frac{\partial S}{\partial a} = -2 \sum_{i=1}^N x_i (y_i - ax_i) = 0$$

$$a = \frac{\sum_{i=1}^N x_i y_i}{\sum_{i=1}^N x_i^2} = \frac{\overline{xy}}{\overline{x^2}}$$

$$(\sigma_a)^2 = \sum_{i=1}^N \left(\frac{\partial a}{\partial y_i} \right)^2 \sigma_i^2 = \frac{\sum_{i=1}^N x_i^2}{\left(\sum_{i=1}^N x_i^2 \right)^2} \varepsilon_y^2 = \frac{\varepsilon_y^2}{N \overline{x^2}} \quad \Rightarrow \quad \boxed{\sigma_a = \frac{\varepsilon_y}{\sqrt{N \overline{x^2}}}}$$

Exercício 4.3.11

$$\begin{cases} \delta y_i = y_i - f(x_i; \alpha) \approx y_i - f(x_i; a) \\ f(x_i; a) = f_i = ax_i \quad \Rightarrow \quad (\Delta f_i)^2 = x_i^2 \sigma_a^2 = \frac{\varepsilon_y^2}{N} \frac{x_i^2}{x^2} \end{cases}$$

$$\begin{cases} (\Delta y_i)^2 = \varepsilon_y^2 + (\Delta f_i)^2 = \varepsilon_y^2 \left(1 + \frac{x_i^2}{N \overline{x^2}} \right) \\ \Delta y_i = \frac{\varepsilon_y}{\sqrt{N}} \sqrt{N + \frac{x_i^2}{\overline{x^2}}} \end{cases}$$

$$\boxed{y_i = \underbrace{ax}_y \pm \underbrace{\frac{\varepsilon_y}{\sqrt{N}} \sqrt{N + \frac{x_i^2}{\overline{x^2}}}}_{\Delta y}}$$