

## THE COSMIC MICROWAVE BACKGROUND SPECTRUM AND AN UPPER LIMIT FOR FRACTAL SPACE DIMENSIONALITY

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### ABSTRACT

The possibility of constraining fractal space dimensionality from astrophysics and other areas is briefly reviewed. Assuming such dimensionality to be  $3 + \epsilon$ , a bound for  $\epsilon$  can be imposed from data obtained by far-infrared absolute spectrophotometer instrument aboard *Cosmic Background Explorer* satellite. The available data for the cosmic microwave background radiation (CMBR) spectrum are fitted by Planck's radiation distribution generalized to noninteger space dimensionality. The present analysis shows that the shape of the CMBR spectrum, which does not depend on the absolute normalization, is correctly described from this distribution provided the absolute temperature is equal to  $(2.726 \pm 0.003) \times 10^{-2}$  K and  $\epsilon = -(0.957 \pm 0.006) \times 10^{-5}$ . This result for the last parameter is shown to be similar to what was found on a very different spatial scale based on a quantum phenomenon. The value of  $|\epsilon|$  is interpreted as an upper limit for how much space dimensionality could have deviated from three. In other words, this is the maximum fluctuation space dimensionality should have undertaken in a very large spatial and temporal scale compared to that of the decoupling era.

*Key words:* cosmic microwave background

### 1. INTRODUCTION

A modern scientific approach to the problem of dimensionality was introduced by Ehrenfest (1917, 1920), who formulated the question: “How does it become manifest in the fundamental laws of physics that space has three dimensions?” The first investigations concerning general relativity and a heuristic model of a pulsating universe were made by Tangherlini (1963, 1968), who tried to impose constraints to integer space dimensionality by searching for bound stable states of the universe. Following the general idea of Kaluza–Klein, there are presently several higher-dimensional theories which agree with all observations, and the two main versions of  $n$ -dimensional cosmology are reviewed by Randall (2002) and Wesson (2003). A modern and comprehensive survey of dimensionality can be found in Petkov (2007).

Ehrenfest's question can obviously be reversed and we can try to answer the following: “How do the fundamental laws of physics entail space dimensionality?” (Caruso & Moreira 1987, 1997). In their 1997 paper, space dimensionality is taken as an unknown quantity or can be admitted to have a noninteger value  $d = 3 + \epsilon$ , with  $\epsilon$  being a parameter to be experimentally determined. Since the introduction of the concept of fractal dimension by Mandelbrot (1977), this became an interesting possibility to be explored.

### 2. THE FIRST PREDICTIONS

Following the general aforementioned idea, with just one exception, several authors have determined limits for  $|\epsilon|$ , as shown in Table 1, covering a very large length scale from micro to macro cosmos.

Except from the two almost similar constraints concerning the cosmic background radiation (CBR), the upper limits for  $|\epsilon|$  vary 4 orders of magnitude for a typical length scale going from  $10^{-8}$  (atomic scale) to  $10^{20}$  m.

So far as astronomical arguments are used, we see from Table 1 that different data from the motion of Mercury give results of the same order of magnitude, namely,  $|\epsilon| \simeq 10^{-9}$ . A quick inspection of this table shows that the two upper limits on  $|\epsilon|$  which follows from an analysis of the CBR are not so stringent as others. The result  $|\epsilon| < 0.02$  (Grassi et al. 1986) was obtained by comparing the available data at that time for the Relic Radiation against a laboratory blackbody source. The limit  $|\epsilon| \leq 10^{-3}$  (Torres & Herrejón 1989) was achieved estimating the CBR experimental errors for both the radiance and the spectrum to be of the order of 10% (Woody & Richards 1981; Smoot et al. 1983; Meyer & Jura 1984; Uson & Wilkinson 1984). An improvement should be pursued since it was shown by Mather et al. (1990) that the deviation of the shape of the cosmic microwave background radiation (CMBR) spectrum from that of a blackbody can be determined with an accuracy much greater than the measurement of the absolute temperature of the sky. Therefore, we should investigate how the more precise 1996 *Cosmic Background Explorer* (COBE) satellite results (Fixsen et al. 1996) can improve the aforementioned limits.

### 3. THE FIT OF COBE DATA

The far-infrared absolute spectrophotometer (FIRAS) on COBE satellite has been designed to measure extremely small deviations of the CMBR from a blackbody spectrum. However, as all previous experimental results on the frequency spectrum of the Relic Radiation, the COBE data do not provide us with absolute measurements.

This experimental restriction is a consequence of the fact that, at the present moment, the anisotropy of the CMBR is not large enough to provide us with a clear choice of some particular dark region of the sky which could help us with the calibration (Richards 1982). However, there is a hope that, at least for the present generation of spectral measurement apparatus, the *Wilkinson Microwave Anisotropy Probe* (WMAP) data, which provide an accurate enough measurement of the dipole spectrum, could lead to a recalibration of the absolute

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**Table 1**  
First Predictions for Deviations  $|\epsilon|$  From the Integer Value 3 for Space Dimensionality

Reference	Measured Quantity	$ \epsilon $
Zeilinger & Svozil (1985)	Anomalous ( $g - 2$ ) electron factor	$(5.3 \pm 2.5) \times 10^{-7}$
Jarlskog & Ynduráin (1986)	Periastron of Mercury and binary pulsar PSR 1913 + 16	$< 1.5 \times 10^{-9}$
Grassi et al. (1986)	3 K radiation frequency spectrum	$< 0.02$
Schäfer & Müller (1986)	Lamb shift in hydrogen	$< 4 \times 10^{-11}$
	Electron Compton wavelength	$\leq 5 \times 10^{-7}$
	Anomalous magnetic moment of the muon	$< 10^{-5}$
Müller & Schäfer (1986)	Perihelion of Mercury	$\approx 10^{-9}$
Torres & Herrejón (1989)	3 K radiation frequency spectrum	$\leq 10^{-3}$

measurement of the FIRAS *COBE* using this data (D. J. Fixsen 2008, private communication).

Nevertheless, they led to the conclusion that the CMBR agrees with a blackbody spectrum to a high accuracy. The published results (Fixsen et al. 1996) are obtained by a comparison against an almost ideal (emissivity  $\approx 0.98$ ) blackbody source placed inside the satellite and calibrated to a second external blackbody (emissivity  $\approx 0.99997$ ). Unfortunately, there is no way to measure fluxes independently. From the theoretical point of view, this comparison is accomplished by assuming the Planck distribution and presupposing space to be three dimensional.

For a  $d$ -dimensional space, Planck's radiation law for the spectral density energy  $u_\nu$ , as a function of temperature  $T$  and frequency  $\nu$ , generalizes to (Caruso & Oguri 2006)

$$u_\nu = \frac{2(d-1)\pi^{d/2}}{\Gamma(d/2)} \left(\frac{\nu}{c}\right)^d \frac{h}{e^{h\nu/kT} - 1}. \quad (1)$$

For the moment, let us attribute any deviation from the ideal three-dimensional blackbody radiation law to be due to the hypothesis that the number of dimension is actually  $d = 3 + \epsilon$  instead of just 3. There is still another assumption underlying our approach: that photons of the 3 K background retain the dimensional information for the place and time of their creation, so it can be compared to here and now.

Thus, the data from Fixsen et al. (1996), complemented with those given at the site of COBE Collaboration (2008), are fitted, using the MINUIT package running the CERN ROOT program, not directly by Equation (1) but by a function of three parameters ( $N$ ,  $T$ ,  $\epsilon$ )

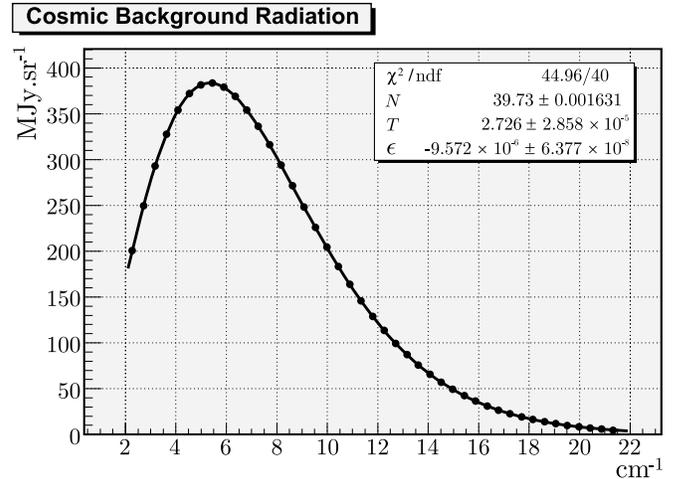
$$u_\nu = N \frac{\nu^d}{e^{h\nu/kT} - 1} \quad (2)$$

since what is relevant to our analysis is the shape of the spectrum. The result of the fit is shown in Figure 1.

The values which emerge from the fit are

$$\begin{cases} \epsilon = -(0.957 \pm 0.006) \times 10^{-5} \\ T = 2.726 \pm 0.003 \times 10^{-2} \text{ K.} \end{cases} \quad (3)$$

Note that the FIRAS experimental result for the absolute temperature of CMBR,  $2.728 \pm 0.004$  K (95% confidence level), found in Fixsen et al. (1996), is dominated by systematic errors. It is important to stress that in our fit we have one more parameter besides a slightly different normalization, which, in practice, did not change the quality of the fit as can be shown by comparing our formal  $\chi^2/\text{dof} = 1.124$  to the 1.15 value found in the *COBE* analysis. Actually, the statistical error to be compared to our result for the absolute temperature is 0.00001, given in



**Figure 1.** Fit of the intensity of the CMBR spectrum as a function of the inverse of wavelength by Planck's radiation law generalized to  $d$  dimensions.

Table 2 of Fixsen et al. (1996). On the other hand, the FIRAS-measured CMBR residuals strongly support the quality of their data and their fit. Thus, it follows that our result is statistically significant as well.

Clearly, the assumption that all deviations from Planck's radiation law might be due to its dependence on dimensionality gives an overestimated value for  $\epsilon$ . Other sources for these deviations indeed exist and were analyzed (Fixsen et al. 1996), such as Bose–Einstein and Compton distortions. In both cases, the upper limits found for the involved parameters indicate a small effect. Based on these results, Wright et al. (1994) estimated that from the Bose–Einstein period ( $10^5 < z < 3 \times 10^6$ ) the ratio between the amount of energy converted from anything but the cosmic background to that of the background itself is  $\lesssim 6.4 \times 10^{-5}$ ; for the energy released after that period but before the decoupling era ( $z \simeq 10^3$ ) the limit is quite similar,  $\lesssim 6.4 \times 10^{-5}$ . Therefore, the value obtained for  $\epsilon$  should also be seen as an upper limit, namely  $|\epsilon| < 0.957 \times 10^{-5}$ , related to the epoch when the universe became entirely transparent: the decoupling era.

#### 4. DISCUSSIONS

Strictly speaking, since absolute measurements for the fluxes are not yet available, the physical meaning of the parameter  $\epsilon$  should be reinterpreted. Indeed, inasmuch as the experimental device compares the background radiation to that of a local blackbody with controlled temperature, at least in principle, one can figure out that the value of  $\epsilon$  could be locally (in this

case, where the reference blackbody is placed) different from the fractal dimensions of space on the horizon scales. In this case,  $\epsilon$  should be interpreted as a difference between these two fractal dimensions at two far away spatial scales, as did by Grassi et al. (1986). Alternatively, one can suppose that local space is just three dimensional and, in this case, we are putting a limit on how much space dimensionality could be different from three in the farthest corner of the universe one can look into.

The overestimated value we get for  $\epsilon$ , Equation (3), is 2 orders of magnitude less than the more accurate upper limit value estimated by Torres & Herrejón (1989) with small errors. Another important difference found here, in respect to this last paper and to that of Grassi et al. (1986), is that we are able to fix also the *sign* of the  $\epsilon$  parameter, showing that it is *negative*. To the best of our knowledge, this is the first time that the sign of  $\epsilon$  can be determined by analyzing experimental results at the astrophysical or cosmological scale. Up to now, the strongest constraint comes from microphysics. Indeed, Zeilinger & Svozil (1985) were able to determine not only the value of  $|\epsilon| = |d - 3|$  but the sign of this parameter too, from quantum field theory, getting  $\epsilon = -(5.3 \pm 2.5) \times 10^{-7}$ , which is approximately 1 order of magnitude less than our upper limit for  $|\epsilon|$ . Their result seems to resolve the discrepancies between the theoretical and experimental values of the anomalous magnetic moment of the electron (see also Svozil & Zeilinger 1986). Another evidence in favor of arbitrarily small nonzero  $|\epsilon|$  comes from the study of Ising gauge theories in noninteger dimensions (Bhanot & Salvador 1986).

In the concluding remarks of their paper, Zeilinger & Svozil (1985) wrote: "it is certainly a challenge for future research to investigate whether or not the deviation of the dimension of spacetime from four can be made more statistically significant than the present work suggests. Furthermore, the question of possible evidence for such a small deviation in other areas of Physics deserves attention." If we assume time to be one

dimensional, as usually done, the main result of the present paper can be seen as an answer to both challenges.

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