

Física Geral - Laboratório

Estimativas e erros em medidas indiretas:
Propagação de erros



Experimentos de *medidas indiretas*

Medidas diretas: Estimativa do valor esperado de uma grandeza a partir de experimentos em que as medidas são lidas diretamente em uma escala, ou registradas por um dispositivo

Medidas indiretas: A estimativa do valor esperado de uma determinada grandeza é obtida a partir da medição (direta) de outras grandezas associadas

Medidas indiretas

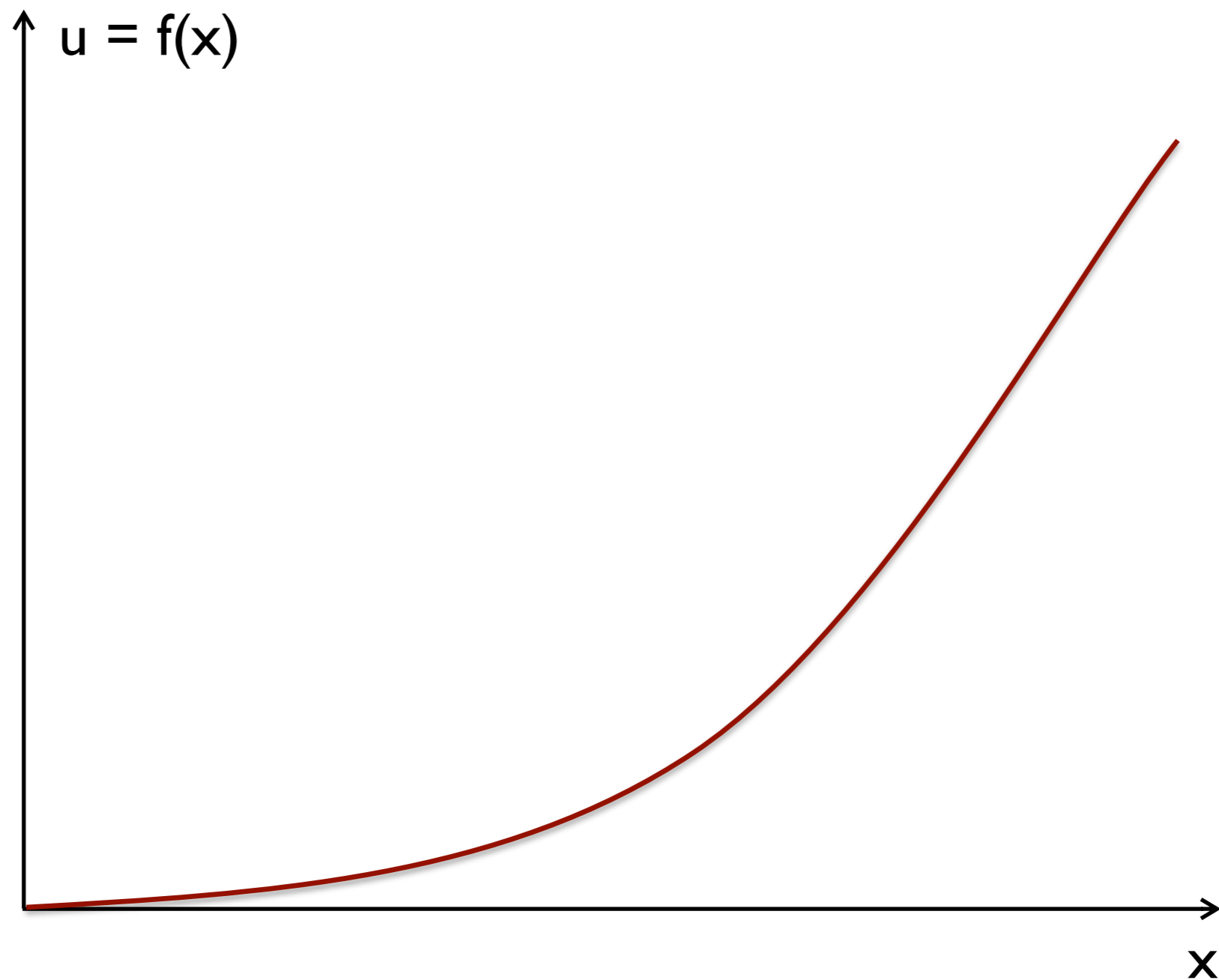
□ Propagação de erros

$$u = f(x)$$

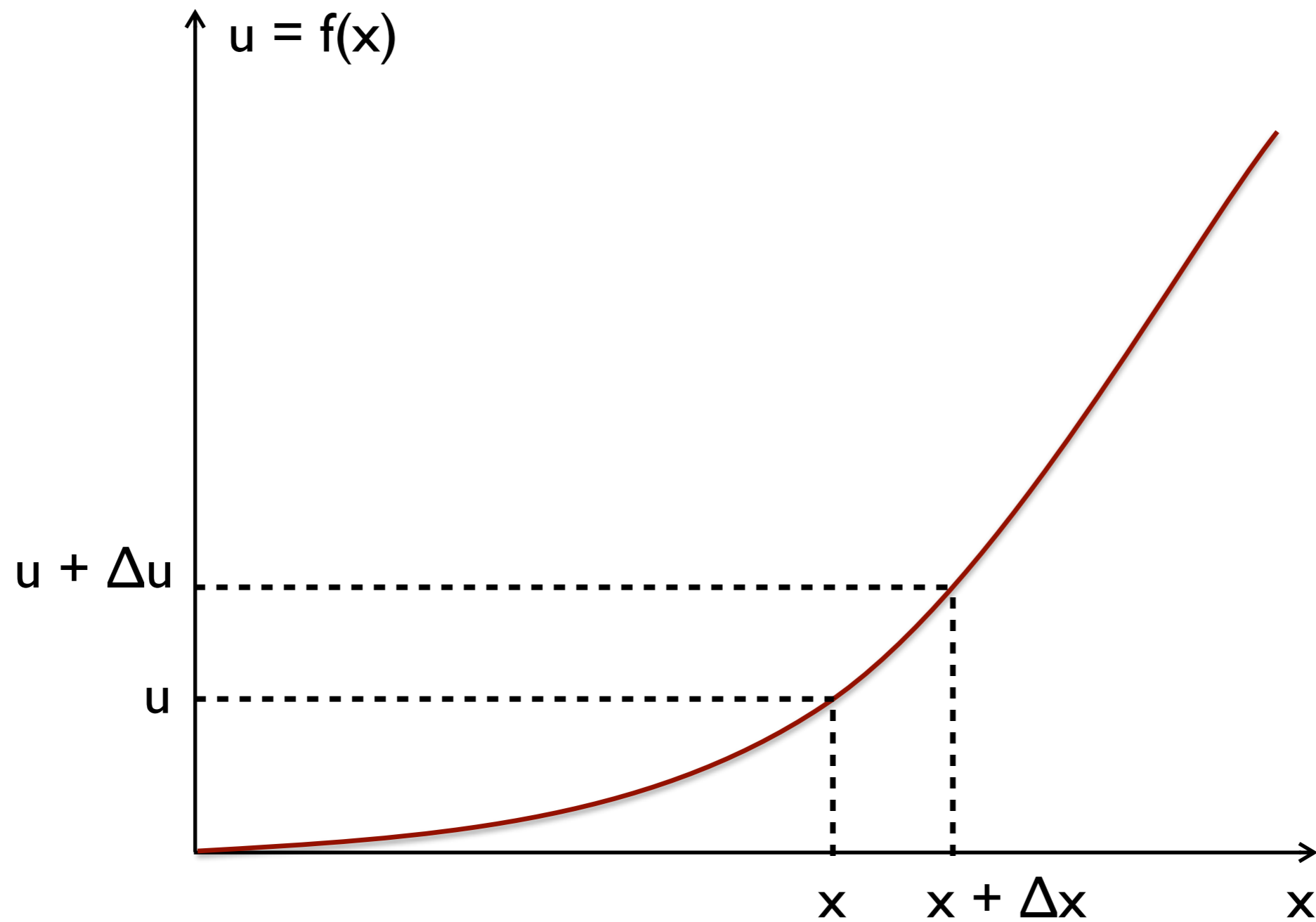
Estimativa da grandeza associada (medida indireta)

Medidas diretas de uma grandeza x :
 $\{x_1, x_2, \dots, x_N\}$

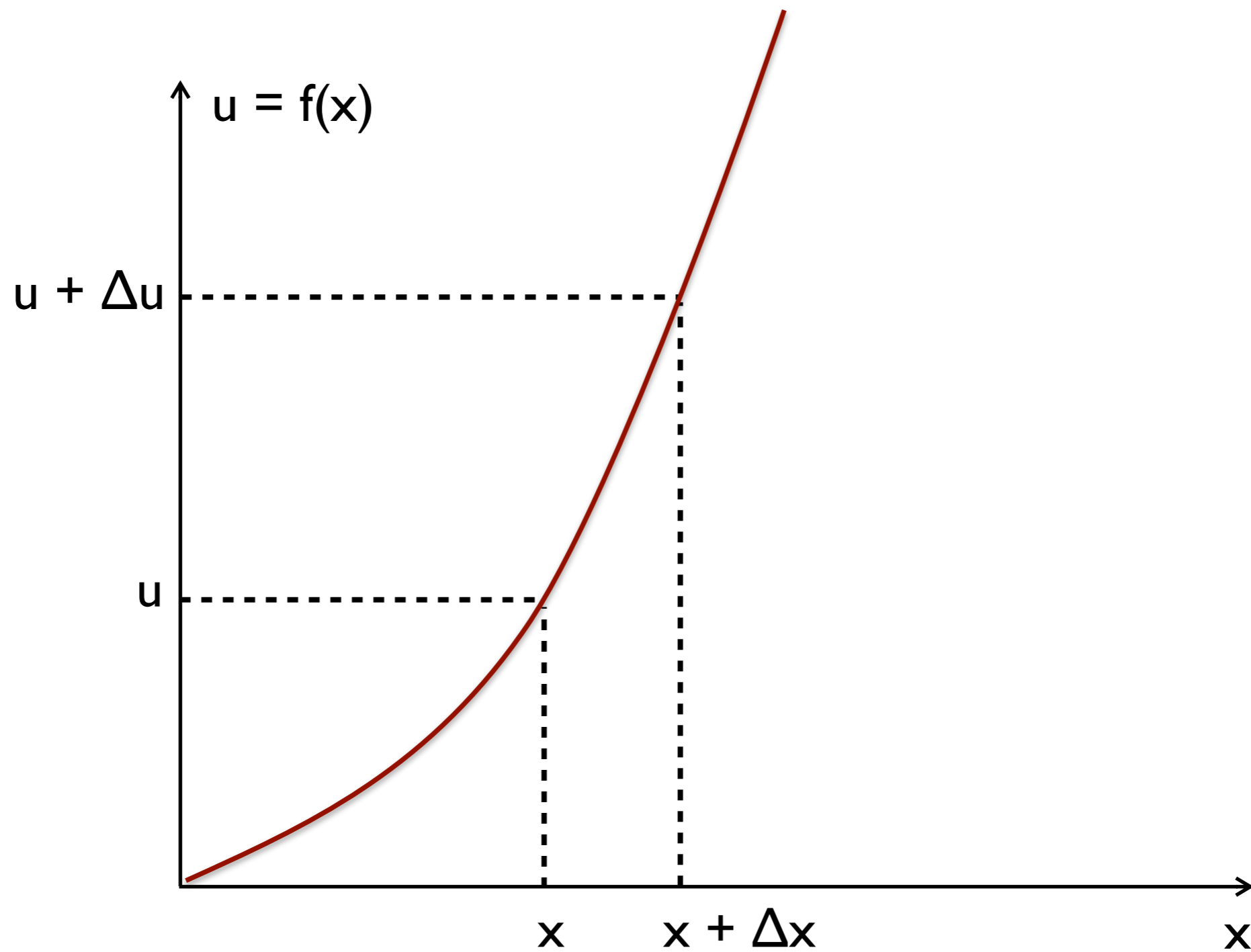
Medidas indiretas - Propagação de erros



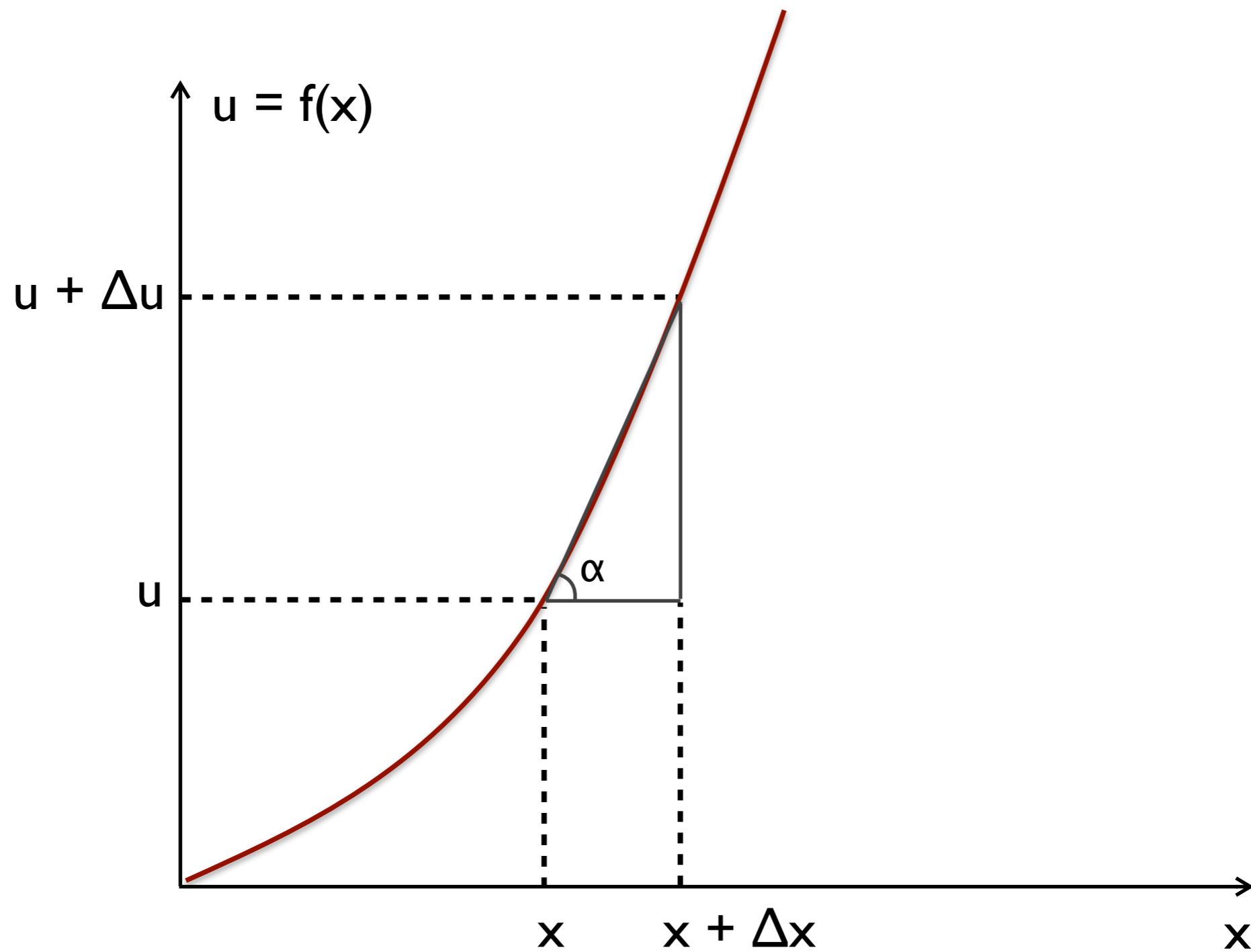
Medidas indiretas - Propagação de erros



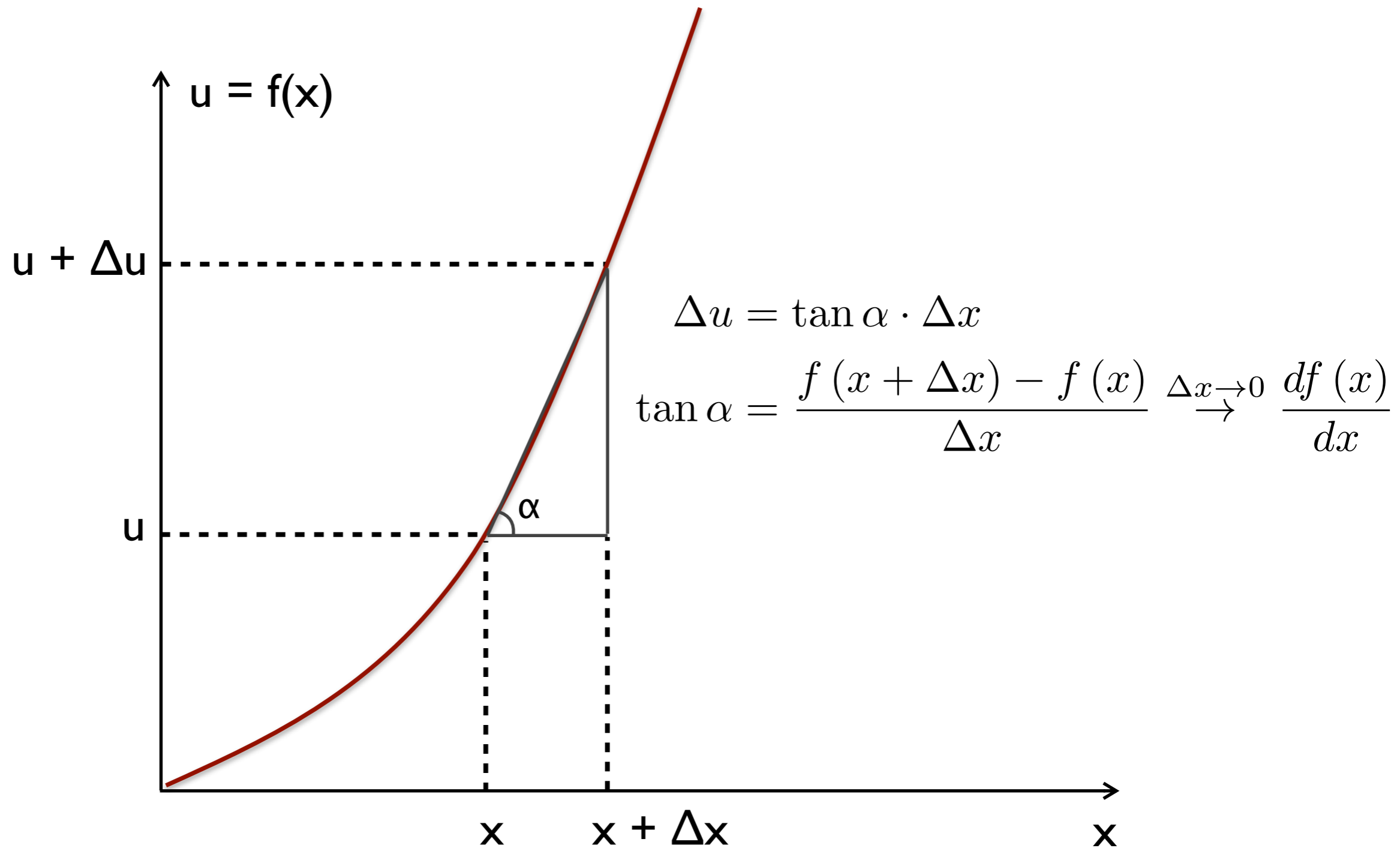
Medidas indiretas - Propagação de erros



Propagação de erros

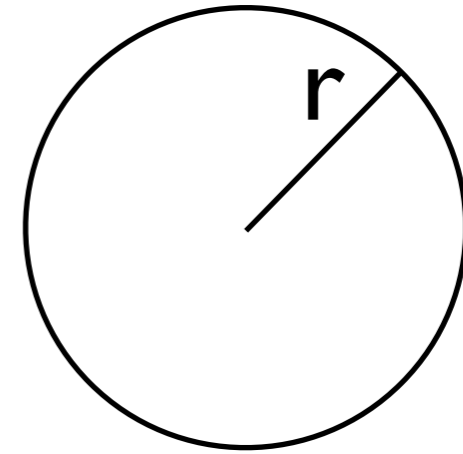


Propagação de erros



Exercício (4.3.3)

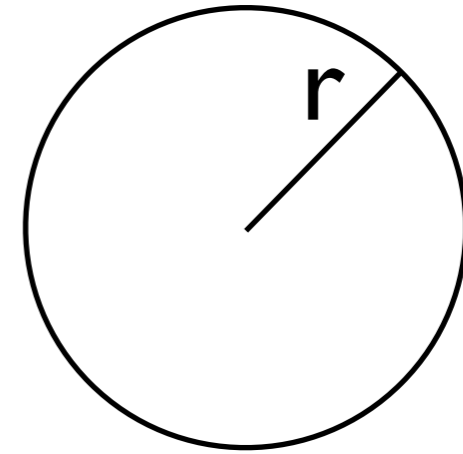
$$A = \pi r^2$$



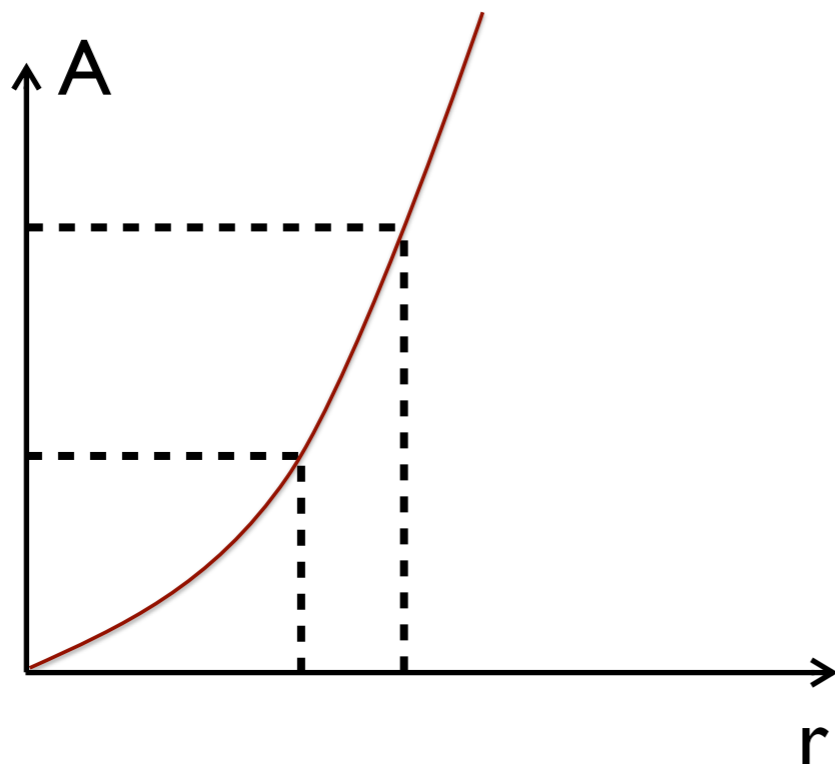
$$r = (10,2 \pm 0,3) \text{ cm}$$

Exercício (4.3.3)

$$A = \pi r^2$$



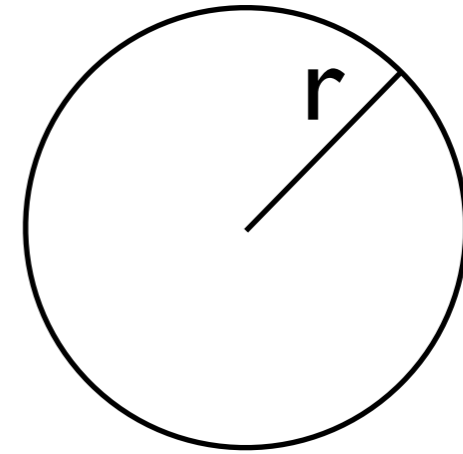
Método aproximado



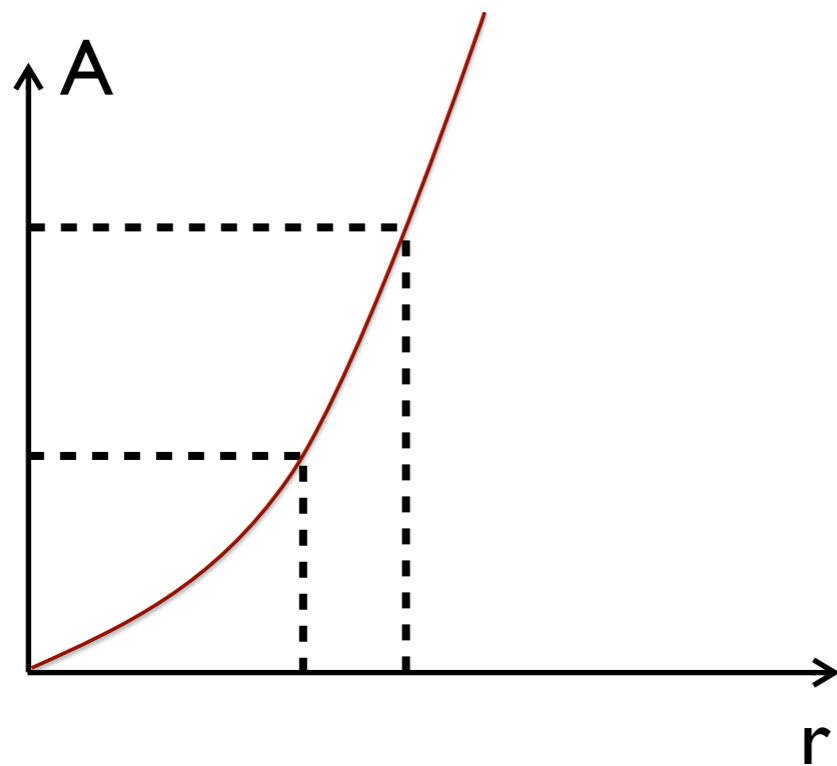
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Exercício (4.3.3)

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Método aproximado

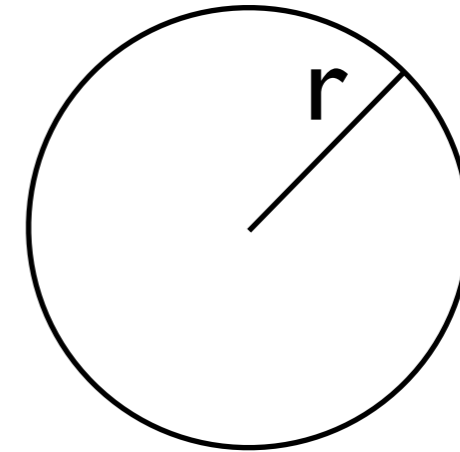


$$r = (10,2 \pm 0,3) \text{ cm}$$

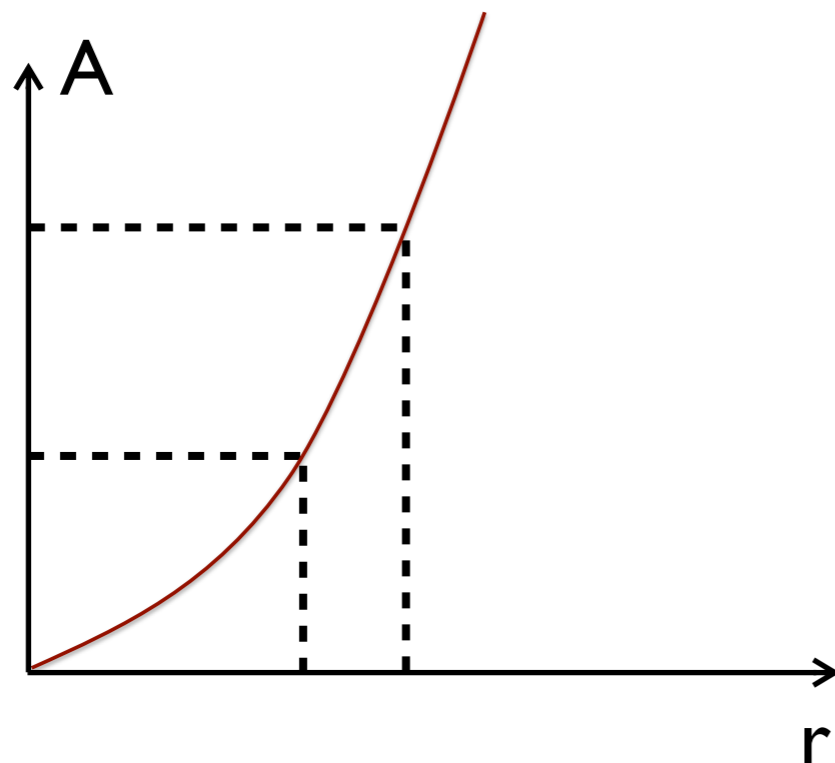
$$A \approx 326,9 \text{ cm}^2$$

Exercício (4.3.3)

$$A = \pi r^2$$



Método aproximado



$$r = (10,2 \pm 0,3) \text{ cm}$$

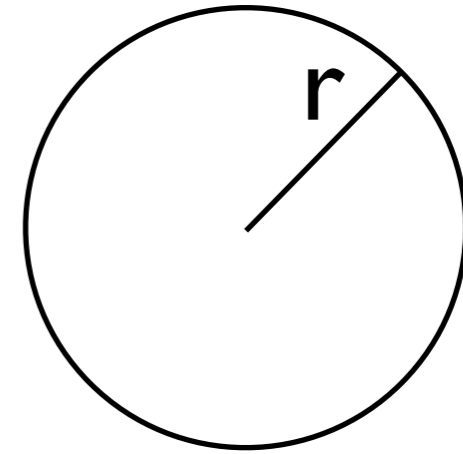
$$A \approx 326,9 \text{ cm}^2$$

$$A^+ \approx 346,4 \text{ cm}^2$$

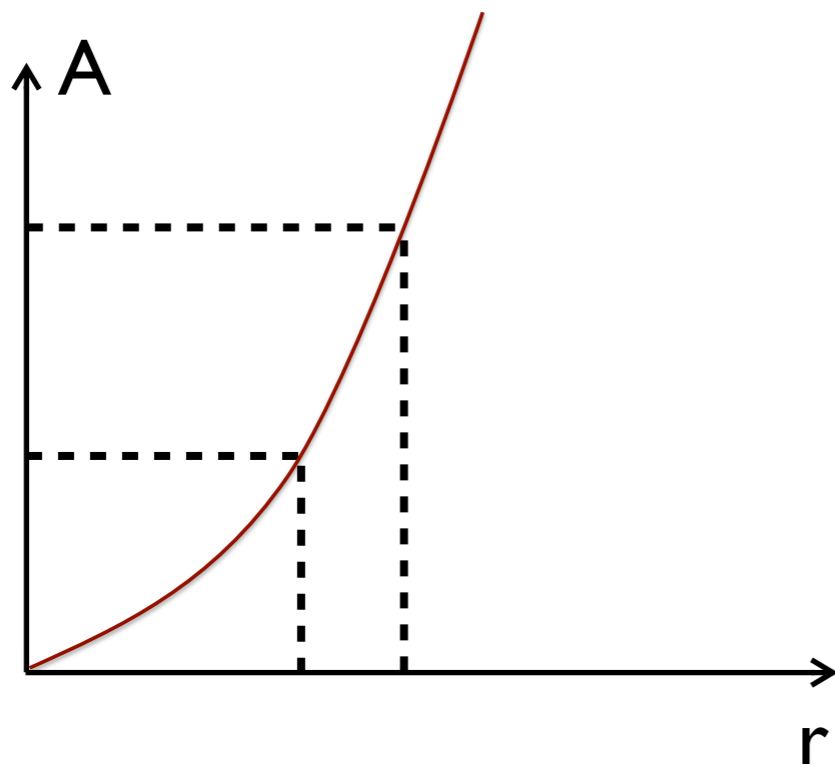
$$A^- \approx 307,9 \text{ cm}^2$$

Exercício (4.3.3)

$$A = \pi r^2$$



Método aproximado



$$r = (10,2 \pm 0,3) \text{ cm}$$

$$A \approx 326,9 \text{ cm}^2$$

$$A^+ \approx 346,4 \text{ cm}^2$$

$$A^- \approx 307,9 \text{ cm}^2$$

$$A = (327 \pm 19) \text{ cm}^2$$

Propagação de erros

$$u = x^2 = f(x)$$

$$u = (\bar{x} + (x - \bar{x}))^2 = (\bar{x} + \Delta x)^2$$

$$u = \bar{x}^2 + 2\bar{x}(x - \bar{x}) + \Delta x^2$$

$$u \approx \bar{x}^2 + 2\bar{x}(x - \bar{x})$$

$$\Rightarrow \bar{u} \approx \bar{x}^2$$

$$u^2 \approx (\bar{x}^2)^2 + 4\bar{x}^2(x - \bar{x}) + 2\bar{x}^3(x - \bar{x})$$

$$\Rightarrow \overline{u^2} \approx \bar{u}^2 + 4\bar{x}^2\sigma_x^2$$

$$\Rightarrow \sigma_u^2 = 4\bar{x}^2\sigma_x^2 \Leftrightarrow \sigma_u = 2|\bar{x}|\sigma_x$$

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$$\overline{(x - \bar{x})} = \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x}) = 0$$
$$\overline{(x - \bar{x})^2} = \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2 = \sigma_x^2$$

Propagação de erros

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$$\sigma_u = \left| \frac{df}{dx} \right|_{x=\bar{x}} \sigma_x = 2|\bar{x}|\sigma_x$$

$$\overline{(x - \bar{x})} = \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x}) = 0$$

$$\overline{(x - \bar{x})^2} = \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2 = \sigma_x^2$$

Propagação de erros

$$u = f(\bar{x}) + \Delta u \approx f(\bar{x}) + \left. \frac{df(x)}{dx} \right|_{x=\bar{x}} (x - \bar{x})$$

$$u^2 \approx [f(\bar{x})]^2 + \left. \left(\frac{df}{dx} \right)^2 \right|_{x=\bar{x}} (x - \bar{x})^2 + 2f(\bar{x}) \left. \frac{df}{dx} \right|_{x=\bar{x}} (x - \bar{x})$$

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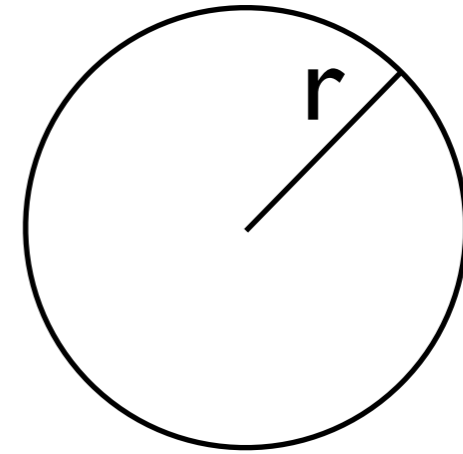
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Exercício (4.3.3)



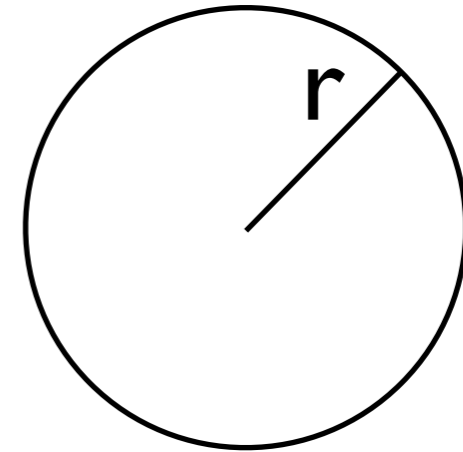
$$r = (10,2 \pm 0,3) \text{ cm}$$

Exercício (4.3.3)

$$A = \pi r^2$$

$$\sigma_{r^2} = 2r\sigma_r$$

$$\sigma_{\overline{A}} = 2\pi\overline{r}\sigma_{\overline{r}}$$



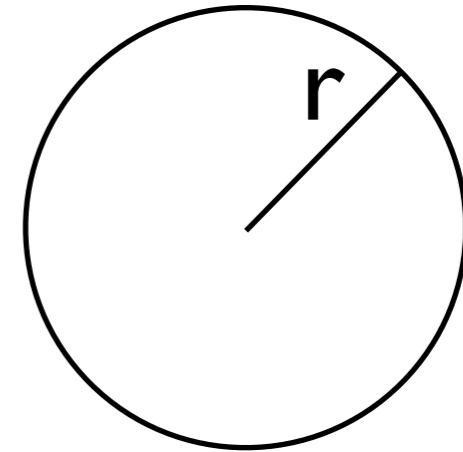
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$$r = (10,2 \pm 0,3) \text{ cm}$$

$$A = (327 \pm 19) \text{ cm}^2$$

Propagação de erros

□ Estimativa padrão da incerteza

$$u = \alpha x \Rightarrow \sigma_{\bar{u}} = |\alpha| \sigma_{\bar{x}}$$

$$u = \alpha x^2 \Rightarrow \sigma_{\bar{u}} = 2 |\alpha| |\bar{x}| \sigma_{\bar{x}}$$

$$u = \frac{\alpha}{x} \Rightarrow \sigma_{\bar{u}} = \frac{|\alpha|}{\bar{x}^2} \sigma_{\bar{x}}$$

Propagação de erros

Exemplo:

$$P = Ri^2$$

$$\Rightarrow \sigma_{\bar{P}} = 2R\bar{i}\sigma_{\bar{i}}$$

Medidas indiretas - Propagação de erros

□ Propagação de erros

$$u = f(x, y)$$

Estimativa da grandeza associada (medida indireta)

Medidas de duas grandezas x e y :
 $\{(x_1, y_1), (x_2, y_2), \dots, (x_N, y_N)\}$

Medidas indiretas - Propagação de erros

□ Propagação de erros

$$u = f(x, y)$$

Estimativa da grandeza associada (medida indireta)

Medidas de duas grandezas x e y :

$$\{(x_1, y_1), (x_2, y_2), \dots, (x_N, y_N)\}$$

Queremos obter: $\bar{u} \pm \sigma_{\bar{u}}$

Propagação de erros

$$\bar{u} = f(\bar{x}, \bar{y})$$

Propagação de erros

$$\bar{u} = f(\bar{x}, \bar{y})$$

Exemplo: $u = x + y$

$$\Rightarrow \bar{u} = \bar{x} + \bar{y}$$

Propagação de erros

$$\bar{u} = f(\bar{x}, \bar{y})$$

Exemplo: $u = x + y$

$$\Rightarrow \bar{u} = \bar{x} + \bar{y}$$

$$u = x/y$$

$$\bar{u} = \bar{x}/\bar{y}$$

Propagação de erros

$$u = x \pm y = f(x, y)$$

$$u = (\bar{x} \pm \bar{y}) + (x - \bar{x}) \pm (y - \bar{y})$$

$$\Rightarrow \bar{u} = (\bar{x} \pm \bar{y}) = f(\bar{x}, \bar{y})$$

$$u^2 = [f(\bar{x}, \bar{y})]^2 + (x - \bar{x})^2 + (y - \bar{y})^2 \pm 2(x - \bar{x})(y - \bar{y}) + 2f(\bar{x}, \bar{y})[(x - \bar{x}) \pm (y - \bar{y})]$$

$$\Rightarrow \overline{u^2} = [f(\bar{x}, \bar{y})]^2 + \sigma_x^2 + \sigma_y^2 \pm 2\sigma_{xy}$$

$$\overline{(x - \bar{x})(y - \bar{y})} = \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})(y_i - \bar{y}) = \sigma_{xy}$$

$$\Rightarrow \sigma_u^2 = \sigma_x^2 + \sigma_y^2 \pm 2\sigma_{xy} \quad \Bigg| \quad \sigma_{\bar{u}}^2 = \sigma_{\bar{x}}^2 + \sigma_{\bar{y}}^2 \pm \frac{2}{N}\sigma_{xy}$$

Propagação de erros

$$u = xy = f(x, y)$$

$$u = [\bar{x} + (x - \bar{x})] [\bar{y} + (y - \bar{y})] = (\bar{x} + \Delta x) (\bar{y} + \Delta y)$$

$$u = \bar{x}\bar{y} + \bar{y}(x - \bar{x}) + \bar{x}(y - \bar{y}) + \Delta x\Delta y$$

$$u \approx \bar{x}\bar{y} + \bar{y}(x - \bar{x}) + \bar{x}(y - \bar{y})$$

$$\Rightarrow \bar{u} \approx \bar{x}\bar{y} = f(\bar{x}, \bar{y})$$

$$u^2 \approx [f(\bar{x}, \bar{y})]^2 + \bar{y}^2 (x - \bar{x})^2 + \bar{x}^2 (y - \bar{y})^2 + 2\bar{y}\bar{x} (x - \bar{x})(y - \bar{y}) + 2f(\bar{x}, \bar{y}) [\bar{y}(x - \bar{x}) + \bar{x}(y - \bar{y})]$$

$$\Rightarrow \overline{u^2} \approx \bar{u}^2 + \bar{y}^2 \sigma_x^2 + \bar{x}^2 \sigma_y^2 + 2\bar{y}\bar{x} \sigma_{xy}$$

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Propagação de erros

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$$r = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

$$\frac{\sigma_u^2}{\bar{u}^2} = \frac{\sigma_x^2}{\bar{x}^2} + \frac{\sigma_y^2}{\bar{y}^2} + 2 \frac{\sigma_{xy}}{\bar{x}\bar{y}} = \frac{\sigma_x^2}{\bar{x}^2} + \frac{\sigma_y^2}{\bar{y}^2} + 2r \frac{\sigma_x \sigma_y}{\bar{x}\bar{y}}$$

Propagação de erros

$$u = x/y = f(x, y)$$

$$u = \frac{\bar{x} + (x - \bar{x})}{\bar{y} + (y - \bar{y})} = \frac{(\bar{x} + \Delta x)}{(\bar{y} + \Delta y)}$$

$$u \approx (\bar{x} + \Delta x) \frac{1}{\bar{y}} \left(1 - \frac{\Delta y}{\bar{y}}\right)$$

$$u \approx \frac{\bar{x}}{\bar{y}} + \frac{1}{\bar{y}} (x - \bar{x}) - \frac{\bar{x}}{\bar{y}^2} (y - \bar{y})$$

$$\Rightarrow \bar{u} \approx \frac{\bar{x}}{\bar{y}} = f(\bar{x}, \bar{y})$$

$$u^2 \approx [f(\bar{x}, \bar{y})]^2 + \frac{1}{\bar{y}^2} (x - \bar{x})^2 + \frac{\bar{x}^2}{\bar{y}^4} (y - \bar{y})^2 - 2 \frac{\bar{x}}{\bar{y}^3} (x - \bar{x})(y - \bar{y}) +$$
$$+ 2f(\bar{x}, \bar{y}) \left[\frac{1}{\bar{y}} (x - \bar{x}) - \frac{\bar{x}}{\bar{y}^2} (y - \bar{y}) \right]$$

$$\Rightarrow \overline{u^2} \approx \bar{u}^2 + \frac{1}{\bar{y}^2} \sigma_x^2 + \frac{\bar{x}^2}{\bar{y}^4} \sigma_y^2 - 2 \frac{\bar{x}}{\bar{y}^3} \sigma_{xy}$$

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$$\frac{1}{1+x} \approx 1-x \quad (x \ll 1)$$

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$$\frac{\sigma_u^2}{\bar{u}^2} = \frac{\sigma_x^2}{\bar{x}^2} + \frac{\sigma_y^2}{\bar{y}^2} - 2 \frac{\sigma_{xy}}{\bar{x}\bar{y}} = \frac{\sigma_x^2}{\bar{x}^2} + \frac{\sigma_y^2}{\bar{y}^2} - 2r \frac{\sigma_x \sigma_y}{\bar{x}\bar{y}}$$

Propagação de erros

□ Em geral:

$$u = f(x, y) \approx f(\bar{x}, \bar{y}) + \left. \frac{\partial f}{\partial x} \right|_{(\bar{x}, \bar{y})} (x - \bar{x}) + \left. \frac{\partial f}{\partial y} \right|_{(\bar{x}, \bar{y})} (y - \bar{y})$$

Propagação de erros

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↓
Derivada parcial de $f(x, y)$ em função de x , com y constante, aplicada no ponto (\bar{x}, \bar{y})

Propagação de erros

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↓
Derivada parcial de $f(x, y)$ em função de x , com y constante, aplicada no ponto (\bar{x}, \bar{y})

$$\Rightarrow \bar{u} \approx f(\bar{x}, \bar{y})$$

Propagação de erros

$$\begin{aligned} \overline{u^2} \approx [f(\bar{x}, \bar{y})]^2 + 2f(\bar{x}, \bar{y}) & \left[\left. \frac{\partial f}{\partial x} \right|_{(\bar{x}, \bar{y})} \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x}) + \left. \frac{\partial f}{\partial y} \right|_{(\bar{x}, \bar{y})} \frac{1}{N} \sum_{i=1}^N (y_i - \bar{y}) \right] \\ & + \left(\left. \frac{\partial f}{\partial x} \right|_{(\bar{x}, \bar{y})} \right)^2 \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2 + \left(\left. \frac{\partial f}{\partial y} \right|_{(\bar{x}, \bar{y})} \right)^2 \frac{1}{N} \sum_{i=1}^N (y_i - \bar{y})^2 \\ & + 2 \left(\left. \frac{\partial f}{\partial x} \right|_{(\bar{x}, \bar{y})} \right) \left(\left. \frac{\partial f}{\partial y} \right|_{(\bar{x}, \bar{y})} \right) \left[\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})(y_i - \bar{y}) \right] \end{aligned}$$

$$\Rightarrow \overline{u^2} \approx [f(\bar{x}, \bar{y})]^2 + \left(\left. \frac{\partial f}{\partial x} \right|_{(\bar{x}, \bar{y})} \right)^2 \sigma_x^2 + \left(\left. \frac{\partial f}{\partial y} \right|_{(\bar{x}, \bar{y})} \right)^2 \sigma_y^2 + 2 \left(\left. \frac{\partial f}{\partial x} \right|_{(\bar{x}, \bar{y})} \right) \left(\left. \frac{\partial f}{\partial y} \right|_{(\bar{x}, \bar{y})} \right) \sigma_{xy}$$

$$\Rightarrow \sigma_u^2 \approx \left(\left. \frac{\partial f}{\partial x} \right|_{(\bar{x}, \bar{y})} \right)^2 \sigma_x^2 + \left(\left. \frac{\partial f}{\partial y} \right|_{(\bar{x}, \bar{y})} \right)^2 \sigma_y^2 + 2 \left(\left. \frac{\partial f}{\partial x} \right|_{(\bar{x}, \bar{y})} \right) \left(\left. \frac{\partial f}{\partial y} \right|_{(\bar{x}, \bar{y})} \right) \sigma_{xy}$$

Propagação de erros

Em geral: $u = f(x, y)$

$$\sigma_{\bar{u}}^2 = \left(\frac{\partial f}{\partial x} \right)^2 \Big|_{(\bar{x}, \bar{y})} \sigma_{\bar{x}}^2 + \left(\frac{\partial f}{\partial y} \right)^2 \Big|_{(\bar{x}, \bar{y})} \sigma_{\bar{y}}^2 + \frac{2}{N} \left(\frac{\partial f}{\partial x} \right) \left(\frac{\partial f}{\partial y} \right) \Big|_{(\bar{x}, \bar{y})} \sigma_{xy}$$

Propagação de erros - Resumo

Exemplo: Adição ou subtração de variáveis

$$u = x \pm y \longrightarrow \sigma_{\bar{u}}^2 = \sigma_{\bar{x}}^2 + \sigma_{\bar{y}}^2 \pm \frac{2}{N} \sigma_{xy}$$

$$\sigma_{\bar{u}} = \sqrt{\sigma_{\bar{x}}^2 + \sigma_{\bar{y}}^2 \pm \frac{2}{N} \sigma_{xy}} \quad \text{ou} \quad \sigma_{\bar{u}} = \sqrt{\sigma_{\bar{x}}^2 + \sigma_{\bar{y}}^2 \pm 2r \sigma_{\bar{x}} \sigma_{\bar{y}}}$$

Propagação de erros - Resumo

Exemplo: Adição ou subtração de variáveis

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Se x e y são *independentes*
(correlação nula)

$$\longrightarrow \sigma_{\bar{u}} = \sqrt{\sigma_{\bar{x}}^2 + \sigma_{\bar{y}}^2}$$

Propagação de erros - Resumo

Exemplo: Multiplicação ou divisão de variáveis

Se x e y são *independentes* (correlação nula):

$$u = xy \quad \longrightarrow \quad \frac{\sigma_{\bar{u}}}{|\bar{u}|} = \sqrt{\left(\frac{\sigma_{\bar{x}}}{\bar{x}}\right)^2 + \left(\frac{\sigma_{\bar{y}}}{\bar{y}}\right)^2}$$

ou

$$u = x/y$$

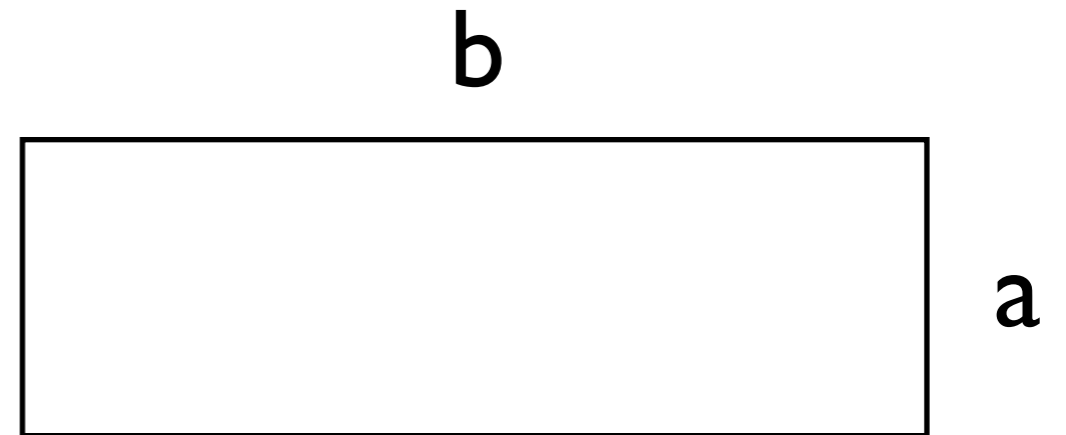
Se a correlação
não é nula:

$$\frac{\sigma_{\bar{u}}}{|\bar{u}|} = \sqrt{\left(\frac{\sigma_{\bar{x}}}{\bar{x}}\right)^2 + \left(\frac{\sigma_{\bar{y}}}{\bar{y}}\right)^2 \pm 2r \left(\frac{\sigma_{\bar{x}}}{\bar{x}}\right) \left(\frac{\sigma_{\bar{y}}}{\bar{y}}\right)}$$

Exercício (4.3.1)

Lado a: $10,32 \pm 0,05$ cm

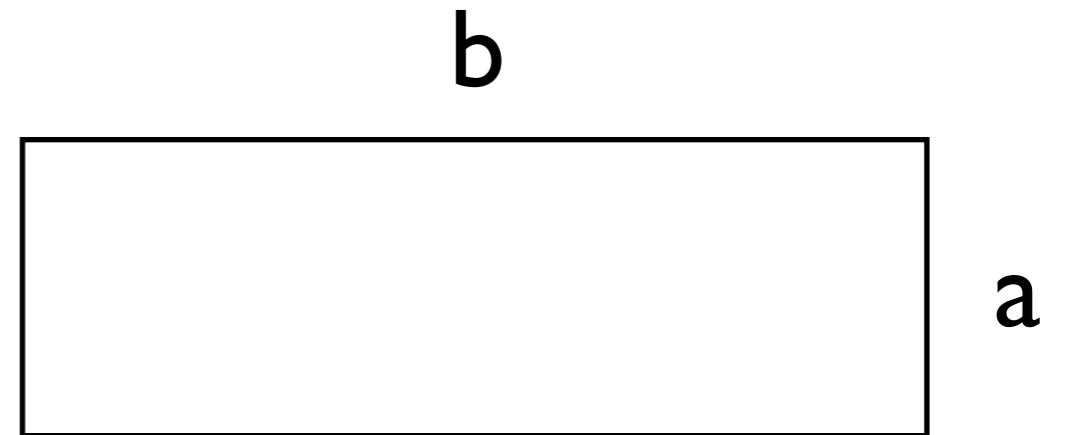
Lado b: $64,27 \pm 0,05$ cm



Exercício (4.3.1)

Lado a: $10,32 \pm 0,05$ cm

Lado b: $64,27 \pm 0,05$ cm



Perímetro: $149,18 \pm 0,14$ cm

Área: $663,27 \pm 3,25$ cm²

Propagação de erros

Exercícios:

i) $u = x^2$

ii) $u = (x \cdot y) / (x + y)$

iii) $u = x + y + z$

iv) $u = xy + z$

v) $p = kl$

vi) $I = V/R$

vii) $v = \sqrt{2gh}$

viii) $T = 2\pi \sqrt{\frac{l}{g}}$

Extras